Saigon International College

Saigon International College
Department of Mathematics and Science
Semester 1, 2022
Year 11 Mathematics Methods ATAR
Test 1

Section One (Calculator Free)

17

Time Allowed : (5+25) minutes **Total marks available:** 35

Name of student: .. Chu . Minh . Dũng ... 11A

Attempt all questions.

Question 1 **(8 marks)**
Solve the following equations for x.

(a) $x^2 + 24x - 25 = 0.$ (2 marks)

$(x+25)(x-1) = 0$ ①

$\therefore x = -25$

or $\therefore x = 1$ ①

(b) $(x-2)^2 - 1 = x + 3$. (3 marks)

$x^2 - 4x + 4 - 1 = x + 3$

$x^2 - 4x + 3 = x + 3$

$x^2 - 7x = 0$

$(x-7)x = 0$

$\therefore x = 0$

$x = 7$

(c) $x^3 + 4x^2 + x - 6 = 0.$ → factors of -6 (3 marks)

± 6
± 1
± 3
± 2

$$\begin{array}{r|rrrr} -1 & 1 & 4 & 1 & -6 \\ & & -1 & -3 & 2 \\ \hline & 1 & 3 & -2 & \text{no} \end{array}$$

$$\begin{array}{r|rrrr} -2 & 1 & 4 & 1 & -6 \\ & & -2 & -4 & 6 \\ \hline & 1 & 2 & -3 & 0 \end{array}$$

$(x+2)(x^2+2x-3)$

$(x+2)(x+3)(x-1)$

$\therefore x = -2$

$\therefore x = -3$

$\therefore x = 1$ ③

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Question 2 **(5 marks)**

The point (3, 4) is the midpoint of point (−1, 5) and point S.

(a) Determine the coordinates of point S. (2 marks)

$\frac{-1+x}{2} = 3$

$-1+x = 6$

$x = 7$

$\frac{5+y}{2} = 4$

$5+y = 8$

$y = 8-5$

$y = 3$

(7, 3)

2

(b) Determine the equation of the straight line that passes through point (2, −1) and is perpendicular to the line through points R and M. (3 marks)

M(3,4) R(-1,5) S(7,3)

~~y = mx + c~~

$y - y_1 = m(x - x_1)$

$5-4 = m(-1-3)$

$1 = m - 4$

$m = \frac{1}{-4}$

$\boxed{y = \frac{1}{-4}x + c}$

$3-5 = m(7+1)$

$-2 = m(8)$

$m = -\frac{2}{8}$

$m = -\frac{1}{4}$

$y = 4x + c$

$-1 = 4(2) + c$

$-1 = 8 + c$

$-1-8 = c$

$-\frac{1}{4}$ x mat the other line = -1

$\frac{-1}{-0.25} = 4$ 1

answer

$y = 4x + c$

$-1 = 4 \times (2) + c$

$-1 = 8 + c$

$\boxed{c = -9} \rightarrow \boxed{y = 4x - 9}$ 1

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Question 4 **(4 marks)**

State the domain and corresponding range for the following functions.

(a) $F(x) = 5 + x^2$

$-\frac{5}{24}$ $\frac{0}{2} = 0$

$y = x^2 + 5$

R ~~domain~~: $\{y \mid y \geq 5; y \in R\}$

D. ~~range~~: $\{x \mid x \in R\}$

(b) $G(x) = \sqrt{x-4}$

range

~~domain~~: $\{y \mid y \geq 0, y \in R\}$ 1

domain: $\{x \mid x \geq 4, x \in R\}$ 1

Question 5

Consider the

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Question 5 **(4 marks)**

Consider the line 2x + by = c where c is a constant.

(a) Find b if the line has gradient - 4. (2 marks)

$2x + by = c$

$by = -2x + c$

$y = \frac{-2x}{b} + \frac{c}{b}$

gradient: $\frac{-2}{b} = -4$

$\frac{-2}{-4} = b$

$\boxed{b = 0.5}$

1

1

(b) Find the c if this line has an X-intercept of 6. (2 marks)

$y = 0.5x + c$

$0 = 0.5(6) + c$

$0 = 3 + c$

$\boxed{-3 = c}$

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0.012 × 20 =

24 | 20
4 0.12

$0.012 \times 20 = 24$

Question 6 **(7 marks)**

(a) The variable P is inversely proportional to the variable t, so that when $t = 2.4$, $P = 20$.

(i) Explain how P will change as t decreases. (1 mark)

$tP^{-1} = \frac{t}{P} = \frac{2.4}{20} = \cancel{0.012}$ 0.12

P will ~~change~~ increase by a factor of 0.12

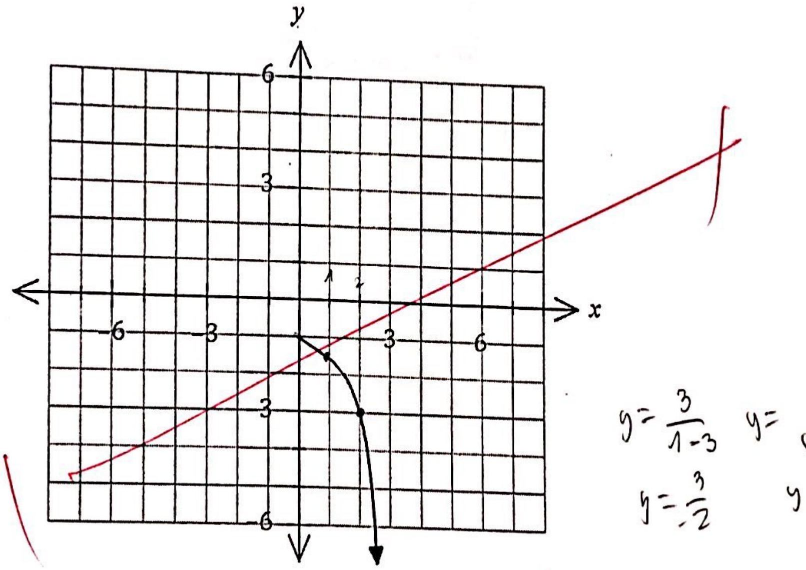
(ii) Determine t when $P = 6$. (2marks)

$\frac{t}{6} = 0.12$

$t = \frac{0.12}{6}$

$t = 0.02$

(c) Part of the graph of $y = \frac{a}{x-3}$ is drawn below.



$y = \frac{3}{1-3}$ $y = \frac{-3}{0-3}$

$y = \frac{3}{-2}$ $y = 1$

$y = \frac{3}{x-3}$

1. (i) Determine the value of a. (1 mark)

$y = \frac{a}{x-3}$

$-3 = \frac{a}{2-3}$ $-3 = \frac{a}{-1}$ $a = -3 \times -1$

$\boxed{a = 3}$

2. (ii) Draw the remainder of the graph. (3 marks)

End of section one

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Department of Mathematics and Science
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Test 1
(Functions and Graphs)

Section Two (Calculator Assumed) 35

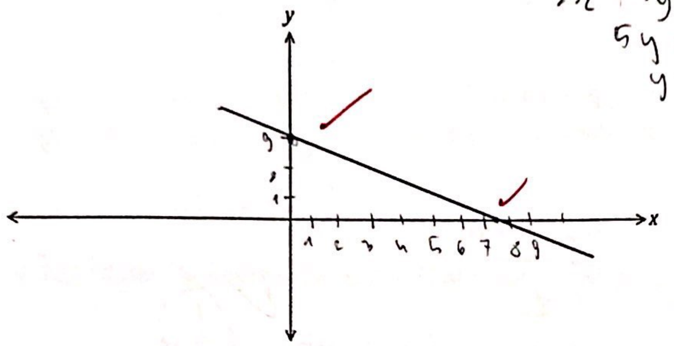
Time Allowed: (5 + 55) minutes **Total Mark available: 51 marks**

Student's Name: Chu Minh Dũng 11A

Question 7 **(6 marks)**

The variables x and y are related by the equation $2x + 5y = 15$.

(a) Sketch the graph of this relationship. (2 marks)



$2x + 5y = 15$
$5y = -2x + 15$
$y = -\frac{2}{5}x + 3$

2

(b) Express y in terms of x and briefly explain why y is a function of x. (2 marks)

$y = -\frac{2}{5}x + 3$ 1

because it passes the vertical line test, there are no repeated x 1

(c) The domain of x is restricted to $5 \le x < 10$. State the range of y. (2 marks)

range: $\{y \mid -1 < y \le 1\}$

$2 \times (5) + 5y = 15$
$10 + 5y = 15$
$5y = 5$
$y = 1$

$2 \times (10) + 5y = 15$
$20 + 5y = 15$
$5y = -5$
$y = -1$

2

Question 9

(a) The variables x an

(i) Sketch th

Question 8

(6 marks)

(a) The points A and B have coordinates (4 , −6) and (5 , 8) respectively. If B is the midpoint of A and C, determine the coordinates of C. (3 marks)

find x : $\frac{4+x}{2} = 5$

$4+x = 10$

$\boxed{x = 6}$ ✓

find y

$\frac{-6+y}{2} = 8$

$-6+y = 16$

$\boxed{y = 24}$ ✗

1 1

C(6, 24) ✓ ✗

(b) The points D and E have coordinates (5p, -q) and (2q, 3p) respectively, where p and q are constants. Determine the value of p and the value of q if the midpoint of D and E is at (21, 17).

find x

$\frac{5p+2q}{2} = 21$

$5p + 2q = 42$

find y:

✓ 1

$\frac{-q+3p}{2} = 17$

$-q + 3p = 34$

$-q = \cancel{34} -3p + 34$

$q = 3p - 34$

$5(3p-34) + 2q = 42$

$15p - 170 + 2q = 42$

$17p = 212$

$\boxed{p = \frac{212}{17}}$

$5\left(\frac{212}{17}\right) + 2q = 42$

$\frac{1060}{17} + 2q = 42$

$2q = -\frac{346}{17}$ ✗

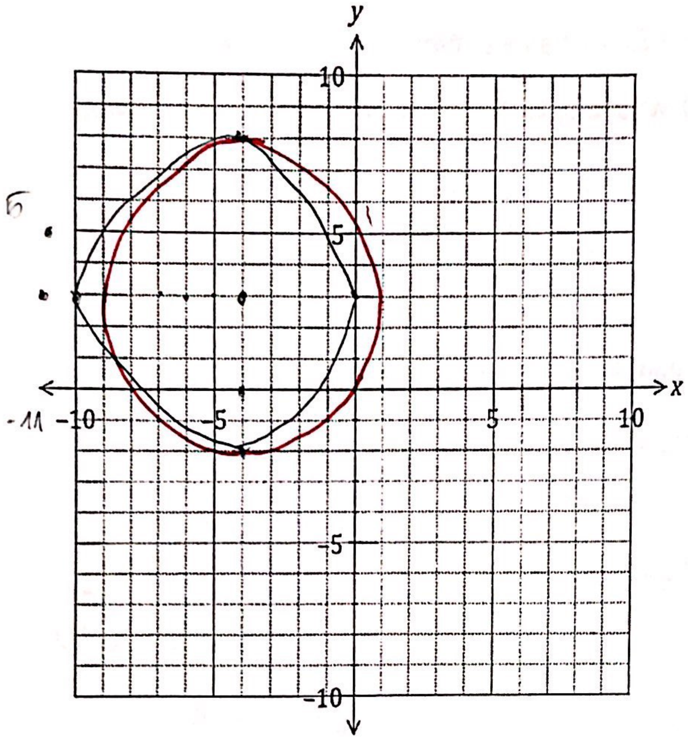
$\boxed{q = -\frac{173}{17}}$

Question 9 **(6 marks)**

$-4 \quad +3$

(a) The variables x and y are related by $(x + 4)^2 + (y - 3)^2 = 25$.

(i) Sketch the graph of this relationship. (3 marks)



1

(ii) How does the vertical line test indicate that y is not a function of x? (1 mark)

The vertide line test intersect the graph two times, so it is not a function ✓ 1

(b) The graph of $(x + 4)^2 + (y - 3)^2 = 25$ that you made in (a) is moved left 7 units and up 2 units. What will be the equation of the graph in its new location? (2 marks)

$(x+11)^2+(y-5)^2=25$

1

Question 10

(6 marks)

The graph $y = f(x)$, where $f(x) = x^2 + bx + c$ has a turning point at $(-2,-1)$.

(a) State the equation of the line of symmetry for the graph of $y = f(x)$. (1 mark)

$x = -2$ 1

(b) Determine the value of the constant b and the value of the constant c. (3 marks)

$y = x^2 + bx + c$

$-1 = 4 + b(-2) + c$

$-1 = 4 - 2b + c$

~~use~~

use turning point form

$y = a(x-b)^2 + c$

$y = (x+2)^2 - 1$ 1

$y = x^2 + 4x + 4 - 1$

$y = x^2 + 4x + 3$

$\boxed{b = 4}$ 1 $\boxed{c = 3}$ 1

(c) The graph of $y = f(x)$ is translated 3 units to the right and 5 units upwards. Determine the equation of the resulting curve. (2 marks)

turning point

$(1, 4)$

$y = (x-1)^2 + 4$ 2

$y = x^2 - 2x + 1 + 4$

$\boxed{y = x^2 - 2x + 5}$

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Question 11 **(8 marks)**

(a) The graph of $y = 2x^2 + bx + 16$ has a line of symmetry with equation $x = 3$.

i. Determine the value of b. (2 marks)

$\cancel{\frac{12}{2\times 2} = 3}$

$\frac{(-)b}{2\times 2} = 3$

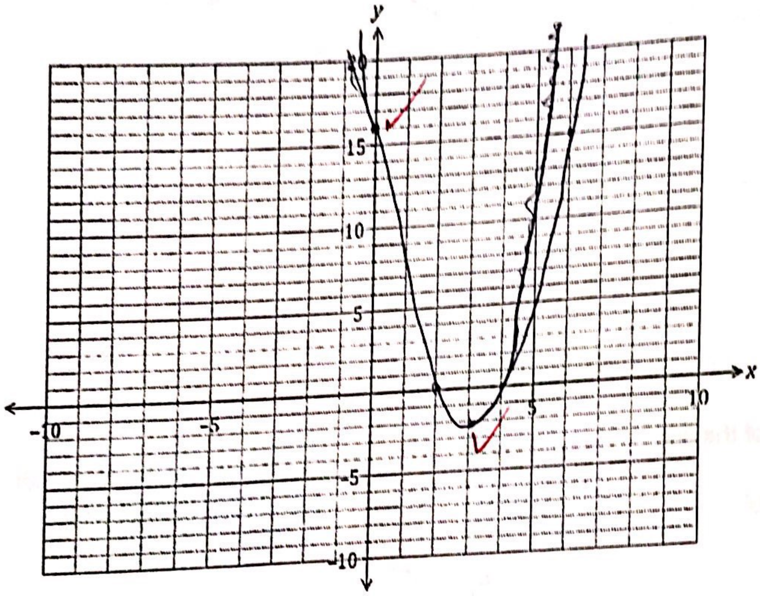
$\frac{-b}{4} = 3$

$-b = 12$

$\boxed{b = -12}$ ✓ (2)

$y = 2x^2 - 12x + 16$

ii. Draw the graph of the parabola on the axis below. (3 marks)



zero: $x = 2$

$x = 4$

$y = 16$

turning point: $(3, -2)$

(3)

(b) One of the solutions to the equation $2x^3 + 21x^2 + cx - 495 = 0$ is $x = 5$. Determine the value of c and all other solutions. (3 marks)

$2(5)^3 + 21(5)^2 + c(5) - 495 = 0$

$250 + 525 + 5c - 495 = 0$

$775 - 495 + 5c = 0$

$280 + 5c = 0$

$-280 = 5c$

$\boxed{c = -56}$ ✓ (1)

$-\frac{9}{2} = 2x$

$2x^3 + 21x^2 - 56x - 495 = 0$

5	2	21	-56	-495
		10	155	495
	2	31	99	0

$-\frac{9}{2}$ (-11)

$(x-5)(2x^2 + 31x + 99)$

$\boxed{(x-5)(2x - \frac{9}{2})(x - 11)} \rightarrow \boxed{x = 5, \; x = -\frac{9}{2}, \; x = -11}$ ✓ (2)

Question 12

(6 marks)

Line L_1 has equation $3x - 4y = 24$.

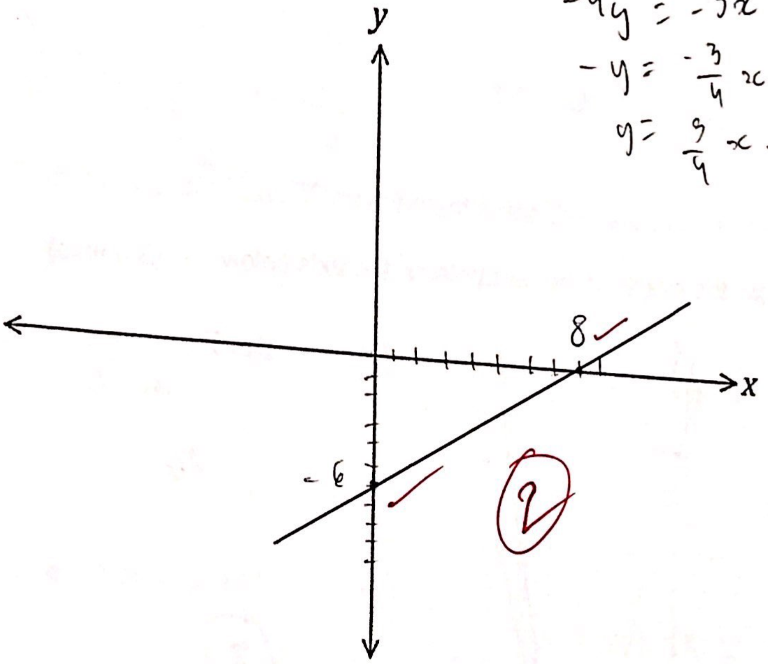
(a) Sketch the graph of L_1.

(2 marks)

$-4y = -3x + 24$

$-y = -\frac{3}{4}x + 6$

$y = \frac{3}{4}x - 6$



y int: -6

x int: 8

$y = mx + c$

(b) Determine the equation of the line L2 that is parallel to L1 and passes through the point with coordinates (-2, -3).

(2 marks)

$y = \boxed{\frac{3}{4}}x - 6$ 1

$y = \frac{3}{4}x - c$

$-3 = \frac{3}{4}(-2) - c$

$-3 = -1.5 - c$

$-1.5 = +c$

$c = -1.5$ 1

$\boxed{y = \frac{3}{4}x - 1.5}$

(c) Determine the equation of the line L3 that is perpendicular to L1 and has the same y intercept as L1.

(2 marks)

y int: -6

$m = -\frac{4}{3}$ 1

$-6 = \frac{4}{3}(0) + c$

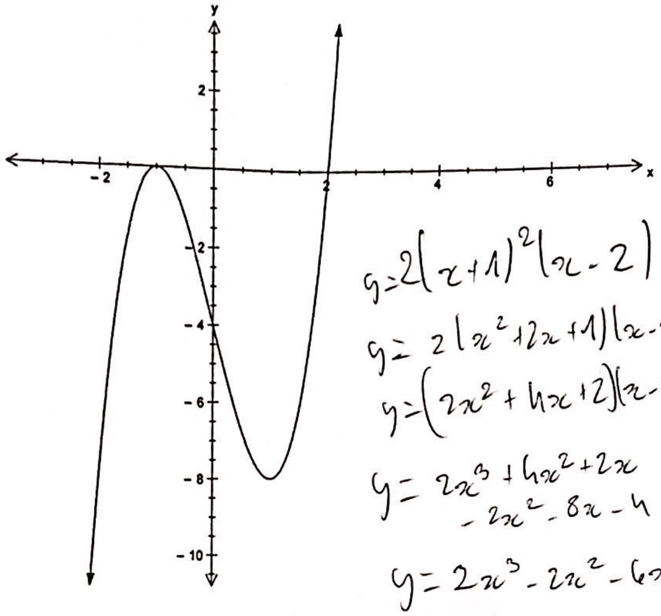
$-6 = c$ 1

$y = -\frac{4}{3}x - 6$

Question 13 **(13 marks)**

(a) The equation of the graph below is $f(x)=ax^3+bx-4$.

(i) Determine the values of a and b. (3 marks)



$y = a(x+1)^2(x-2)$

$-4 = a(1)^2(-2)$

$-4 = a \times (-2)$

$\frac{-4}{-2} = a$

$a = 2$ ✓ 1

$y=2(x+1)^2(x-2)$

$y=2(x^2+2x+1)(x-2)$

$y=(2x^2+4x+2)(x-2)$ $\cancel{y=2x}$

$y=2x^3+4x^2+2x-2x^2-8x-4$

$y=2x^3-2x^2-6x-4$

$y=2x^3+bx-4$

$\cancel{4=2x^3+bx-4}$

(ii) Use the graph above to state the possible k values such that $f(x)=k$ has only 2 solutions. (2 marks)

$k=-1$

$k=2$ ✗

(b) (i) Show that $(x+2)$ is a linear factor of the cubic equation $x^3-3x^2-3x+14=0$. (2 marks)

$$\begin{array}{r|rrrr} -2 & 1 & -3 & -3 & 14 \\ & & -2 & 10 & -14 \\ \hline & 1 & -5 & 7 & 0 \end{array}$$

no remainder ✓

2

(i) Express the cubic in the form $x^3 - 3x^2 - 3x + 14 = (x+2)(x^2 + ax + b)$ evaluating the coefficients a and b (2 marks)

(iii) Hence, state the number of real root(s) of the function $f(x) = x^3 - 3x^2 - 3x + 14$.

Justify your answer using the discriminant, $\Delta = b^2 - 4ac$. (4 marks)

End of Section Two

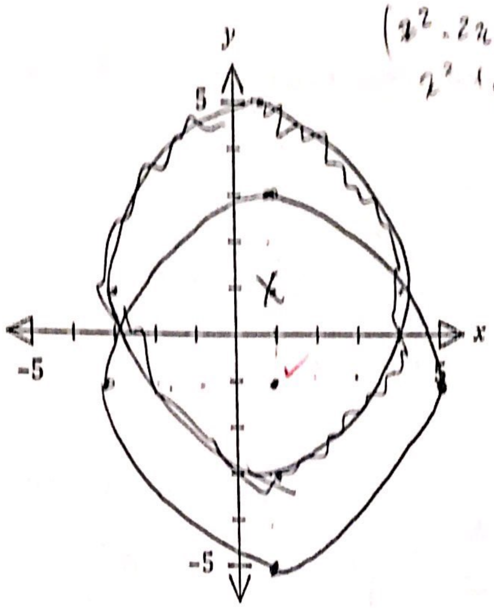
Question 3

(7 marks)

(a) Sketch the graph of $(x-1)^2+(y+1)^2=4$ on the axes below. (3 marks)

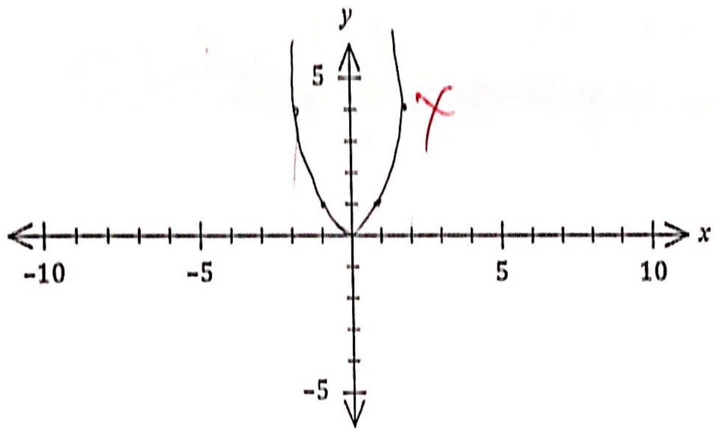
$(x^2-2x+1)+(y^2+2y+1)=4$

$x^2+y^2-2x+2y+2=0$



1 1

(b) Sketch the graph of $y^2=x$ on the axes below. (2 marks)



(c) Explain whether y is a function of x in the relationship graphed in (b). (2 marks)

yes y ~~is~~ is a function of x because $y^2=x$ passes the ~~veptrole~~ vertical line test